Analysis of Discrete Wavelet Spectra of Broadband Signals

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Abstract

The article is devoted to the development of a mathematical model for autoregressive and autocoherent analysis of discrete wavelet spectra with approbation on known experimental data. The model is used to detect anomalies or emergency states of the object after wavelet filtering of noise and zero crossing. The proposed method for processing noisy signals, combining noise filtering and Mahalanobis proximity analysis, shows better machine learning results than neural networks and other methods for detecting classification and recognition anomalies. The wavelet spectra were processed using autoregressive and autocoherent analyses. The possibility of using these functions as classification features for identifying possible anomalous states of devices is shown. Before forming the wavelet spectra, the procedure of direct with filtering and inverse transformation of the already filtered signal is carried out. As a result, it seems possible to identify the characteristic features of the signal - anomalies or emergency conditions. The main factor in the classification of device status signals is the moment when changes in the signal spectrum begin to develop, therefore, all analysis, both in the time and frequency domain, is tied either to the number of samples by the time the signal was received, and in some cases even to the extended date format - month number.

Keywords 1

Wavelet spectrum, autocoherence, autoregression, state anomaly, machine learning

1. Introduction

Computer simulation, processing, identification, and analysis of signals are among the most urgent tasks facing intelligent systems today. The task of analyzing the state signals of complex objects attracts the attention of specialists, since there is no single approach and method for identifying signals in computerized control and monitoring systems. This problem has not been solved for broadband signals, for which the frequency band used for signal transmission is much wider than the minimum required for information transmission.

The most common approach to signal analysis is the search and comparison with a signal database containing signal samples with which the obtained data are compared [1]. Comparison of signal records with a reference base can be carried out in different ways: cross-correlation; distance comparisons; cluster analysis; application of the likelihood ratio; hybrid expert methods [2]. These methods are widely covered in the literature.

A well-known approach to solving this problem of analyzing and classifying signals is to find the optimal feature space in which objects (signals) can most easily be separated using classical classification algorithms [3, 4]. If the diagnostic analysis is carried out for a long period of operation and is characterized by the appearance and development of increasing non-informative noise, the known methods of signal analysis cannot be applied, since the comparison of signals will be incorrect

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[5]. The problem can be solved by analyzing the state signals of devices and identifying the characteristic features of the signal that cause anomalous or emergency states of objects.

The implementation of this approach makes it possible to obtain correct data for the formation of a comparative base of signals for known methods of signal identification, and to become an independent analysis tool. Since in real conditions of monitoring and control it is not uncommon to work with signals about which there are no a priori data [2], the use of classification of status signals significantly expands the scope of diagnostic analysis of signals.

2. Analysis of the literature data and a formulation of the problem

The analysis shows that the most common types of signal errors are: stop of data transmission from the measuring device, atypical data outliers, failures of measuring equipment, accumulation of errors due to interference, data breaks, etc. [6]. In addition, the need for data processing in comparison with the specified threshold signal values also precludes a qualitative analysis of information [6].

The use of filtering the data of the diagnostic analysis of the control object makes it possible to eliminate noise, however, in the future, it is not possible to isolate from the data the signs of loss of device operability. Numerous publications on this problem are devoted to the search for an algorithm or a combination of data processing methods for selective filtering of information.

Under conditions of growth of non-informative noise, diagnostic signals of complex objects are non-stationary complex signals, which are characterized by the presence of complex time dependences of amplitude, frequency, and phase. Therefore, it seems logical to apply filtering using the wavelet transform. In the general case, the wavelet transform is the decomposition of a signal into a wavelet spectrum (signal decomposition) with subsequent processing (detailing) and reconstruction [7-10]. The height of the signal peak and its location in the frequency spectrum are ideal characteristics for classifiers (random forest, gradient boosting, logistic regression, etc.) used in machine learning [11]. However, studies in [8-12] confirm that filtering using only the continuous wavelet transform does not solve the problem of filtering high-frequency and low-frequency noise.

Signal classification using discrete wavelet transform (DWT) is as follows: DWT is used to separate the signal into different frequency subbands [13]. If there are different frequency characteristics in the signals, the characteristic features appear in one of the frequency subranges. Thus, when generating features from each subrange and using a set of features as input data for a classifier in machine learning (random forest, gradient boosting, logistic regression, etc.), the task of identifying state signals for various types of signals is realized [14]. Therefore, classifier machine learning using signal processing methods has wide application prospects.

The literature widely describes the application of various criteria that are used in the application of classical classification algorithms. The work [15] considers the implementation of the block of mathematical signal processing, which is implemented by deconstructing the signal into its frequency subbands, from each subbands areas are generated that can be used as input data for the comparative proximity of the series. Authors [16. 17] use the autocoherence function to determine the local in time non-stationarity of a random process [18-20]. The above model of autocoherence is used both in the spectral and in the correlation representation. This makes it possible to determine signal anomalies in local frequency and time intervals.

Analysis of publications shows that a large amount of experimental data arrays makes it impossible to use a single function for all arrays. For real-time data processing, other approaches should be used, for example, the initial filtering of incoming data by filtering out obviously incorrect measurements [6].Thus, research related to the development and improvement of signal analysis methods for the subsequent application of machine learning methods is an urgent scientific task. The purpose of this work is to develop a mathematical model for autoregressive and autocoherent analysis of discrete wavelet spectra for the classification of noisy signals by machine learning.

3. Mathematical model of discrete wavelet spectra

This paper uses empirical data collected during experimental studies of reliability characteristics using four bearings (B1-B4) on a loaded shaft (6000 pounds) rotating at a constant speed of 2000 rpm

[21]. The 2nd_test set containing one accelerometer was used for the analysis. The dataset is formatted in separate files, each containing a 1-second snapshot of the vibration waveform recorded at specific time intervals. Each file consists of 20480 points with a sampling frequency of 20 kHz [21]. The file name indicates the date of the data capture which occurred every 10 minutes. The known data were taken in order to compare the obtained results on the processing of vibration monitoring data performed by other authors [21].

3.1 Autocoherence for Stationarity Analysis of Discrete Spectra

If there is noise in the signal, then the series can become non-stationary and it is impossible to extract the necessary information from it, which is usually contained in the low-frequency part of the spectrum. Such a characteristic for the series of wavelet coefficients is autocoherence, which can be determined from the well-known relation for the continuous wavelet spectrum:

$$W_X = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} I(t)\varphi\left(\frac{t-b}{a}\right) dt, \tag{1}$$

where $\varphi\left(\frac{t-b}{a}\right)$– wavelet function; W_X– the scale-time spectrum of the signal I(t); a – the scale factor, b – the signal shift along the time axis.

To determine autocoherence, one should use the relation for wavelet coherence:

$$R^2(a,b) = \frac{|s(a^{-1}W_{XY}(a,b))|^2}{s(a^{-1}|W_X(a,b)|^2)s(a^{-1}|W_Y(a,b)|^2)}, \tag{2}$$

where s – smoothing operator [10]. Coherence is interpreted as the square of the correlation coefficient, and its values range from 0 to 1.

Autocoherence is determined by a simple substitution in (2) of relations (1) and (3):

$$W_Y = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \frac{dI(t)}{dt} \varphi\left(\frac{t-b}{a}\right) dt, \tag{3}$$

It should be noted that the relation for autocoherence is used to estimate the stationarity of a number of discrete wavelet coefficients.

3.2 Proximity of time series for the analysis of autoregression and autocoherence

Let us designate the next two time series of the wavelet coefficients of the spectra as:

$$X_i = \{\tilde{a}_{j,k}\}, \{\tilde{d}_{2,k}\}, \dots, \{\tilde{d}_{j,k}\} \dots \{\widetilde{d_{J,k}}\}, k = 1,2, \dots \frac{N}{2^j}, j = 1 \dots J,$$
$$Y_i = \{\tilde{a}_{j,k}\}, \{\tilde{d}_{2,k}\}, \dots, \{\tilde{d}_{j,k}\} \dots \{\tilde{d}_{J,k}\}, k = 1,2, \dots \frac{N}{2^j}, j = 1 \dots J, \tag{4}$$

where X_i – series of wavelet coefficients at time t_i; Y_i – series of wavelet coefficients at time t_{i+dt}; dt – determined by the time for which a change in the spectrum will not lead to equipment failure.

The correlation closeness of series (4) is determined from the relationship:

$$proximity = 1 - \frac{\sum_{i=1}^{N} X_i Y_i}{\sqrt{\sum_{i=1}^{N} X_i^2 \sum_{i=1}^{N} Y_i^2}}. \tag{5}$$

When the spectra series are close and completely identical proximity=0 when there is no difference -proximity=1. On practice $0 \le proximity \le 1$.

3.3. Decomposition of the Time Series

The wavelet coefficients in the decomposition of the measuring signal f(t) are generally determined from the following system of equations [22]:

$$\left.\begin{aligned} a_{j,k} &= \int_R f(t)\varphi_{j,k}(t)dt \\ d_{j,k} &= \int_R f(t)\psi_{j,k}(t)dt \end{aligned}\right\}, \tag{6}$$

where R – the interval for determining the function f(t) of the signal; $a_{j,k}$, $d_{j,k}$ – the coefficients of approximation and detailing of the discrete wavelet decomposition of the signal, respectively; $\varphi_{j,k}$, $\psi_{j,k}$ – father and mother wavelets, respectively; j, k – he current level of the wavelet decomposition and the ordinal number of the wavelet coefficient in the wavelet decomposition of the signal, respectively.

In calculations, instead of integrating (6), Mull's pyramidal algorithm is used to eliminate methodological errors associated with quadrature formulas [23]. The approximating coefficients are set at the conditionally zero level j_o, $j_{0,k}$, k=1,…N, where $N=2^m$, m>1. Accuracy was assessed using the predictive regression metrics method and an uncertainty matrix.

Next, approximating $a_{j_{0+1,k}}$ and detailing $d_{j_{0+1,k}}$ coefficients are calculated, and from $a_{j_{0+1,k}}$ вычисляют $a_{j_{0+1,2\,k}}, d_{j_{0+1,2\,k}}$ and so on. In the presence of noise with zero mathematical expectation and standard deviation, the set of coefficients enclosed in arrays array ({}) for the maximum decomposition level J will take the form of the formula:

$$\{\tilde{a}_{j,k}\},\{\tilde{d}_{2,k}\},\ldots,\{\tilde{d}_{j,k}\}\ldots\{\tilde{d}_{j,k}\}, k = 1,2,\ldots\frac{N}{2^j}, j = 1\ldots J, \tag{7}$$

The set of nested arrays (7) is transformed into a 1D list using the special flatten function [24]:

$$[\tilde{a}_{j,k},\tilde{d}_{2,k},\ldots\tilde{d}_{j,k}\ldots\tilde{d}_{j,k}], k = 1,2,\ldots\frac{N}{2^j}, j = 1\ldots J, \tag{8}$$

3.4 Noise Reduction by Limiting Detail Factors

The filtered signal is determined from the relation [25]:

$$\hat{f}(x) = \sum \bar{a}_{j+j,0,k}\varphi_{j+j_{0,k}}(t) + \sum_{j=1}^{j}\sum_{k}[F(\lambda_i)\bar{d}_{j+j_{0,k}}\psi_{j+j_{0,k}}(t)], \tag{9}$$

where $F(\lambda_j)$ – threshold function from the list garotte, garrote, greater, hard, less, soft, for example, hard(λ_j), λ_j – threshold at the j decomposition level; $\varphi_{j+j_{0,k}}(t), \psi_{j+j_{0,k}}(t)$ – the "father" and "mother" wavelets, respectively [25]. The universal threshold λ_j is determined from the relation [25]:

$$\left.\begin{aligned} \sigma &= \frac{\mathrm{median}(|d_{1,k}|)}{0,6742}, \\ \lambda_j^{univ} &= \sigma\sqrt{2\ln N_j}, \end{aligned}\right\} \tag{10}$$

where λ_j^{univ} – universal threshold; N_j – he number of detail coefficients $d_{1,k}$ at the j decomposition level; *median*($|d_{1,k}|$)– median from the array of detail coefficients.

A common threshold can be used as the average of λ_j^{univ} or both series (4):

$$\lambda_X = \lambda_Y = \frac{1}{n}\sum_{i=1}^{N}\lambda_{i,} \tag{11}$$

The universal threshold $\lambda_j^{univ} = \sigma\sqrt{2\ln N_j}$ of limiting the wavelet coefficients of the spectrum depends on the dispersion and the number of wavelet coefficients of detail, therefore, when analyzing discrete spectra, it is an essential characteristic of high-frequency noise.

Depending on the nature of the signal, additional mathematical processing can be applied to discrete wavelet coefficients [25].

4. Regression Analysis of Real Data Based

Applying the proposed model of the method of regression analysis of discrete spectra, it is possible to clearly identify the date 2004-02-19 06:12:39 of a possible defect for the B3 device (Figure 1). Identification occurs both by the number of accumulated maxima - 10 exceeding the threshold of 0.5 correlation proximity, and by the maximum value.

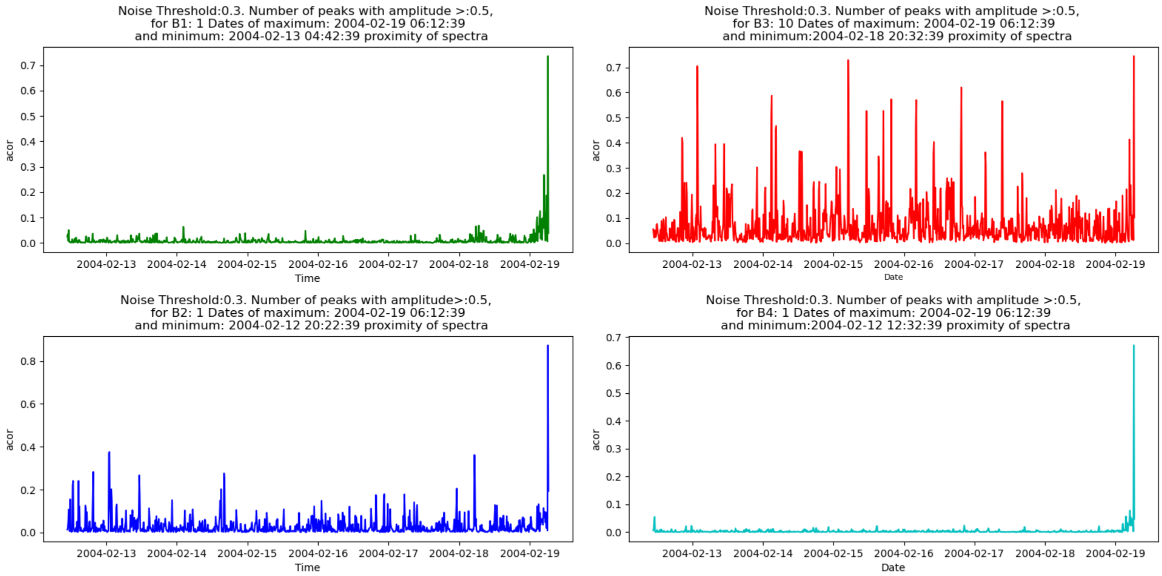


Figure 1: Autoregression of discrete wavelet spectra obtained from relations (4), (5) with filtering by a common threshold (11)

5. Coherent analysis of real data based on the developed mathematical model

The analysis shows that the proximity of the autocoherence of the spectra is the highest - unity for the B3 device and a minimum of autocoherence. Thus, the loss of stationarity is achieved on 2004-02-19 06:12:39 (Figure 2). Not despite the fact that the number of peaks -91 is less than that of the device B2.

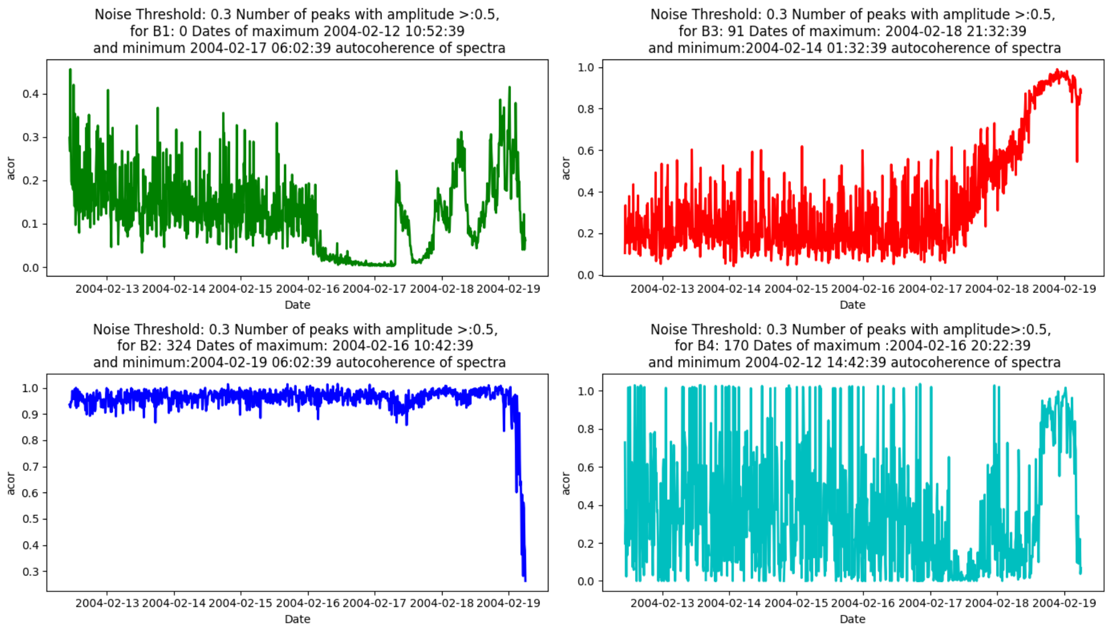


Figure 2: Autocoherence of discrete wavelet spectra obtained by relations (1), (2), (3), (4), (5) with common threshold filtering (11)

6. Using the Mahalanobis algorithm to analyze discrete wavelet spectra

To detect signal anomalies, we use zero-crossing analysis, which has practically not been used in relation to discrete wavelet spectra. This approach is designed to detect chirp and non-linear chirp crossover signals. These include the average value of the derivative, the zero crossing frequency, and the average transition rate of a given signal amplitude [25]. Combined with noise filtering and Mahalanobis proximity analysis, zero-crossing analysis shows better machine learning results than neural networks and other classification and recognition anomaly detection methods.

Features of the Mahalanobis algorithm consist in the use of a covariance matrix according to the relation [26]:

$$D_M(\bar{x}) = \sqrt{(\bar{x} - \bar{\mu}_x)^T S^{-1} (\bar{y} - \bar{\mu}_y)}, \quad (12)$$

where $\bar{x}, \bar{y}$ – multidimensional series of wavelet coefficients; $\overline{\mu_X}, \overline{\mu_Y}$ – mathematical expectations; *S*– he covariance matrix.

The covariance matrix S and its inverse matrix are symmetric and positive definite. Therefore, the transformation of the spectral series of wavelet coefficients from multidimensional to one-dimensional is not required. At the same time, wavelet decomposition into levels during filtering and zero-crossing dramatically increase the number of identification features. The increase in the number of features only due to zero-crossing and filtering can be seen by comparing the graphs of the training sets (Figure 3).

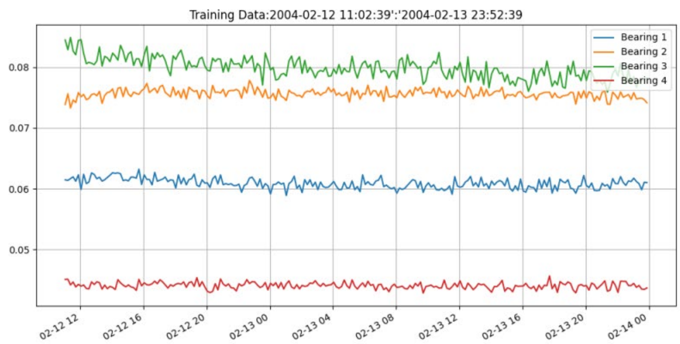


Figure 3: Graph of the training set signals in the trouble-free period of operation without the use of wavelet noise filtering and zero-crossing

From the graph (Figure 4) it follows that the method of regression analysis of discrete spectra clearly identifies the date 2004-02-19 06:12:39 of a possible defect for device B3 both by the number of accumulated maxima– 10 exceeding the threshold of 0.5 correlation closeness, and by the maximum value.

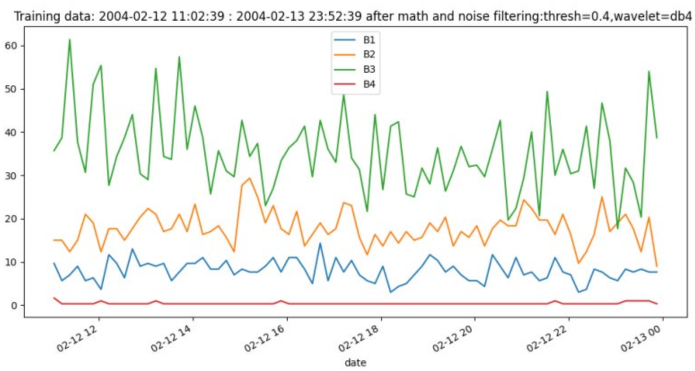


Figure 4: Graph of the training set signals in the trouble-free period of operation using wavelet noise filtering and zero-crossing

The graphs of test set signals also have a significant difference (Figure 5, Figure 6).

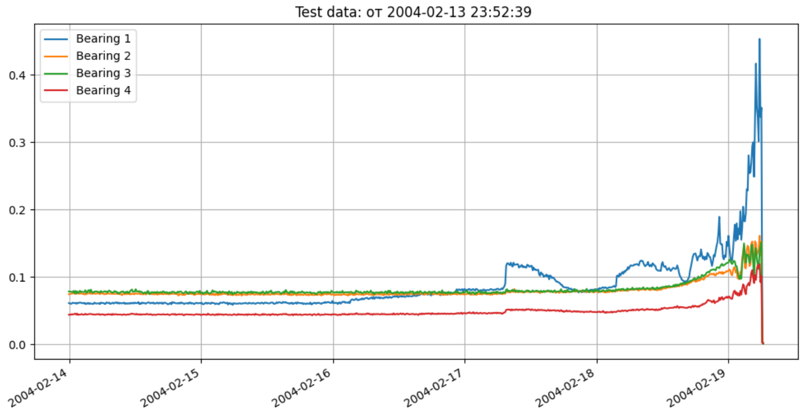


Figure 5: Graph of test set signals during detection of anomalies or emergency conditions without wavelet noise filtering and zero crossing

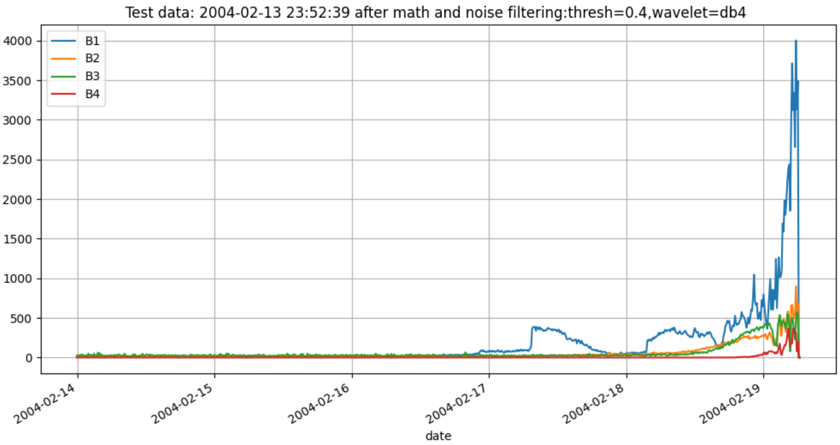


Figure 6: Graph of test set signals in the period of detection of anomalies or emergency conditions after wavelet filtering of noise and zero crossing

When determining anomalies using the described approach, a clear identification of signs of anomalies and emergency conditions of the object is observed, as evidenced by a strong oscillation of the Mahalanobis distance in the threshold zone (Figure 7, Figure 8).

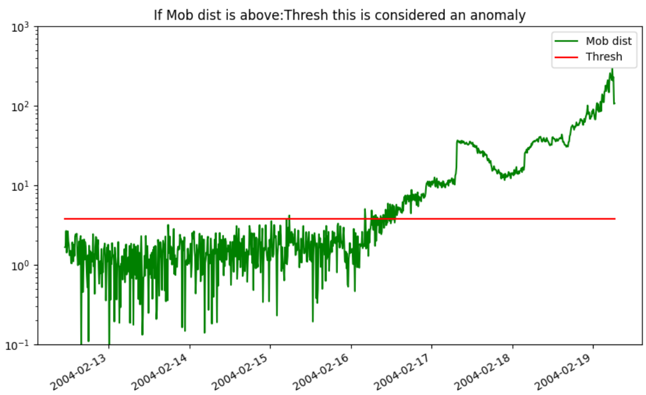


Figure 7: Changing the Mahalanobis distance in the threshold zone without signal processing

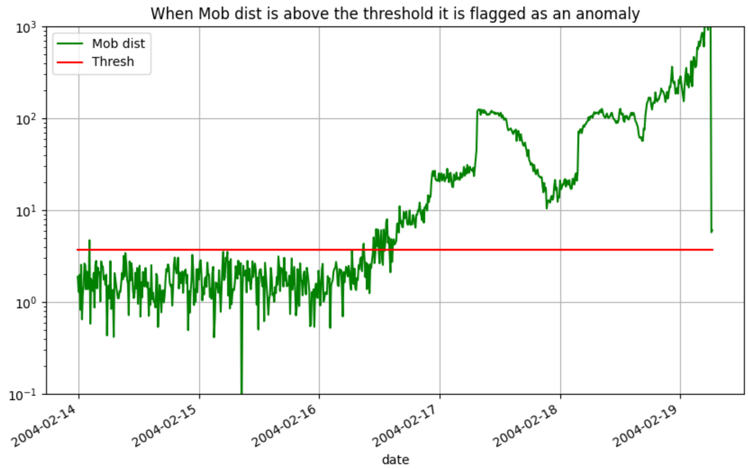


Figure 8: Changing the Mahalanobis distance in the threshold zone using Filtering Wavelet and Zero Crossing

7. The Results of the Classifier Using the Machine Learning Method

To test the proposed method for classifying state signals using the obtained features, the classifiers described by the following algorithms were studied: gradient method, linear (SVM), logistic regression, nearest neighbors, decision tree (Table 1) on the resulting computer model.

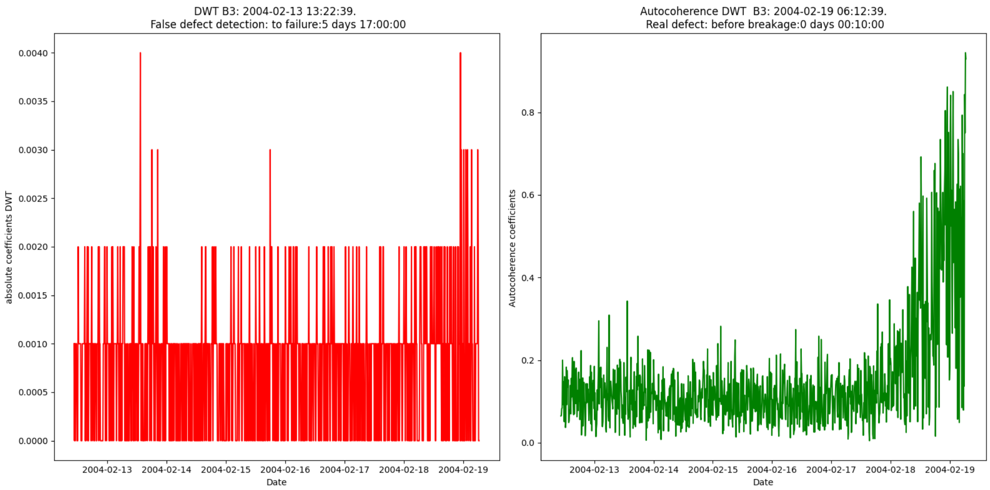
The previously obtained wavelet data analysis model was used for machine learning with the gradient method signal classifier (as the most commonly used). Time series of wavelet coefficients db38 of cosine proximity (autocoherent proximity) were taken as features. The results of machine learning of the model confirm the effectiveness of the proposed algorithm (Table 1).

To confirm the effectiveness of the developed method for classifying noisy signals using the machine learning method, we will process the same signal without using the established signs of state classification (Figure 9, a) and using the developed classifier (Figure 9, b).

Table 1

Efficiency of application of algorithms for machine learning of classifiers

№	Algorithm name	Accuracy on the training sample at the level of detail			Accuracy on the test sample at the level of detail			Studying time with varying degrees of detail		
		0	0,3	0,4	0	0,3	0,4	0	0,3	0,4
1	logistic regression	0,949	0,934	0,916	0,912	0,907	0,920	0,148	0,016	0,023
2	linear (SVM)	1,000	1,000	0,996	0,910	0,904	0,926	0,083	0,057	0,041
3	nearest neighbors	0,931	0,938	0,916	0,899	0,900	0,916	0,045	0,003	0,006
4	gradient method	1.000	1,000	1,000	0,891	0,959	0,905	1,907	1,091	1,964
5	decision tree	1,000	1,000	1,000	0,838	0,950	0,855	0,039	0,024	0,019



a b

Figure 9: Comparative analysis of the results of signal processing without (a) and with (b) the use of the developed method for classifying state signals

The use of noise filtering with its own time-dependent universal threshold makes it possible to increase the accuracy of predicting the occurrence of object defects.

8. Conclusions

1. A new method for classifying noisy signals using machine learning has been developed. Proposed method for processing noisy signals that combines noise filtering and Mahalanobis proximity analysis. The essence of the method lies in the following: the data obtained after mathematical processing for all frequency subranges of the signal and a given moment of time constitute time series, filtering each series from noise with its own, time-dependent, universal threshold. The time series filtered in this way are used as input signals to the machine learning classifier and characterize the change in the state of the control object over time. The use of this classifier implements pre-processing of the signal in order to eliminate noise and allows you to highlight the signs of inoperable states of devices in dynamics.

2. The list of used functions for processing wavelet spectra has been extended with two more ones: by cosine proximity of the autocoherence spectra of discrete wavelet spectra and by the cosine proximity function of wavelet spectra of signals following one after another in time. The possibility of using these functions as classification features for identifying possible anomalous states of devices has been confirmed.

3. A new criterion for classifying the loss of uptime is based on the presence of local minima depending on the sum of squares of the wavelet detailing coefficients at the level of the wavelet decomposition.

4. The developed classification method was tested using a computational experiment on a known data set obtained using an accelerometer, all stages of the classification of an object's inoperable state signal are shown. The obtained results of machine learning using the signal classifier gradient method confirm the effectiveness of the application of new signs of signal classification in wavelet analysis (the accuracy on the test set was 0.96).

9. References

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